LECTURE 2:

Algorithms
pseudocode; examples

Organizational:

Webpage: up and running.

Newsgroup: algouvt on yahoo groups. Please subscribe.

First homework: posted tomorrow on the webpage.

DEADLINE (firm): Friday, October 19, 5pm.

Outline

- Continue with algorithms/pseudocode from last time.
- Describe some simple algorithms

- Decomposing problems in subproblems and algorithms in subalgorithms

Properties an algorithm should have

- Generality
- Finiteness
- Non-ambiguity
- Efficiency

Efficiency

An algorithm should use a reasonable amount of computing resources: memory and time

Finiteness is not enough if we have to wait too much to obtain the result

Example:

Consider a dictionary containing 50000 words.

Write an algorithm that takes a word as input and returns all anagrams of that word appearing in the dictionary.

Example of anagram: ship -> hips

Efficiency

First approach:

Step 1: generate all anagrams of the word

Step 2: for each anagram search for it in the dictionary (using binary search)

Let's consider that:

- the dictionary contains n words
- the analyzed word contains m letters

Rough estimate of the number of basic operations:

- number of anagrams: $m!$
- words comparisons for each anagram: $\log_2 n$ (e.g. binary search)
- letters comparisons for each word: m

$$\boxed{m! * m * \log_2 n}$$

Efficiency

Second approach:

Step 1: sort the letters of the initial word

Step 2: for each word in the dictionary having m letters:

- Sort the letters of this word
- Compare the sorted version of the word with the sorted version of the original word

Rough estimate of the number of basic operations:

- Sorting the initial word needs almost m^2 operations (e.g. insertion sort)
- Sequentially searching the dictionary and sorting each word of length m needs at most nm^2 comparisons
- Comparing the sorted words requires at most nm comparisons

$$\boxed{n\,m^2 + nm + m^2}$$

Efficiency

Which approach is better ?

First approach	Second approach
$m!\ m \log_2 n$	$n\ m^2 + n\ m + m^2$

Example: m=12 (e.g. word algorithmics)
n=50000 (number of words in dictionary)

8* 10^10	8*10^6

one basic operation (e.g.comparison)= 1ms=10^{-3} s

24000 hours	2 hours

Thus, important to analyze efficiency and choose more efficient algorithms

Outline

- Problem solving
- What is an algorithm ?
- Properties an algorithm should have
- Describing Algorithms
- Types of data to use
- Basic operations

How can we describe algorithms ?

Solving problems can usually be described in mathematical language

Not always adequate to describe algorithms because:

- Operations which seem elementary when described in a mathematical language are not elementary when they have to be encoded in a programming language

Example: computing a sum, computing the value of a polynomial

Mathematical description

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$$

Algorithmic description

(it should be a sequence of basic operations)

How can we describe algorithms ?

Two basic instruments:

- Flowcharts:
  - graphical description of the flow of processing steps
  - not used very often, somewhat old-fashioned.
  - however, sometimes useful to describe the overall structure of an application
- Pseudocode:
  - artificial language based on
    - vocabulary (set of keywords)
    - syntax (set of rules used to construct the language’s “phrases”)
  - not as restrictive as a programming language

Why do we call it pseudocode ?

Because …

- It is similar to a programming language (code)
- Not as rigorous as a programming language (pseudo)

In pseudocode the phrases are:

- Statements or instructions (used to describe processing steps)
- Declarations (used to specify the data)

Types of data

Data = container of information

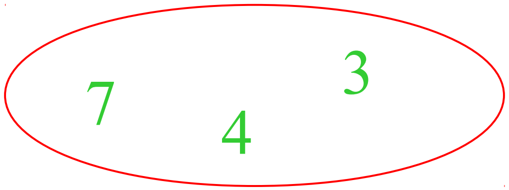
Characteristics:

- name
- value
  - constant (same value during the entire algorithm)
  - variable (the value varies during the algorithm)
- type
  - primitive (numbers, characters, truth values …)
  - structured (arrays)

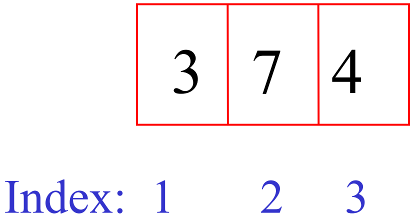
Types of data

Arrays - used to represent:

- Sets (e.g. {3,7,4}={3,4,7})
  - the order of the elements doesn’t matter

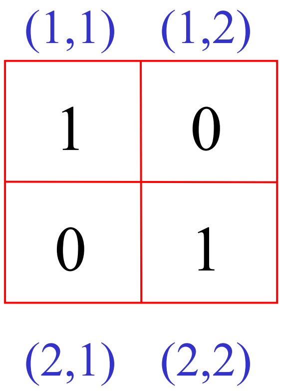


- Sequences (e.g. (3,7,4) is not (3,4,7))
  - the order of the elements matters



- Matrices
  - bidimensional arrays

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



How can we specify data ?

- Simple data:

Integers	INTEGER <variable>
Reals	REAL <variable>
Boolean	BOOLEAN <variable>
Characters	CHAR <variable>

How can we specify data ?

Arrays

One dimensional

<elements type> <name>[n1..n2]

(ex: REAL x[1..n])

Two-dimensional

<elements type> <name>[m1..m2, n1..n2]

(ex: INTEGER A[1..m,1..n])

How can we specify data ?

Specifying elements:

- One dimensional

  x[i] - i is the element’s index

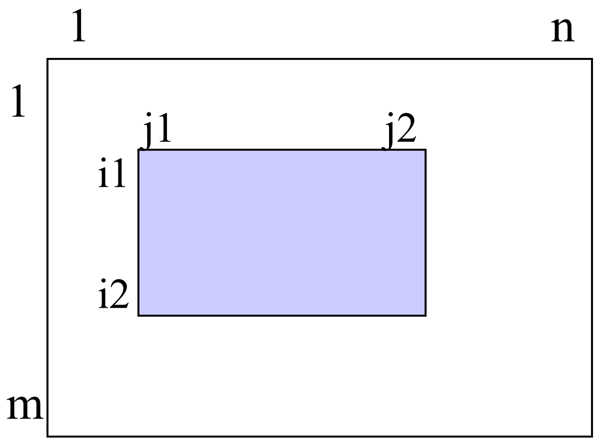
- Two-dimensional

  A[i,j] - i is the row’s index, while j is the column’s index

How can we specify data ?

Specifying subarrays:

- Subarray= contiguous portion of an array
  - One dimensional: x[i1..i2] (1<=i1<i2<=n)
  - Bi dimensional: A[i1..i2, j1..j2]
    (1<=i1<i2<=m, 1<=j1<j2<=n)



Outline

- Problem solving
- What is an algorithm ?
- Properties an algorithm should have
- Describing Algorithms
- Types of data to use
- Basic instructions

What are the basic instructions ?

Instruction (statement)

= action to be executed by the algorithm

There are two main types of instructions:

- Simple
  - Assignment (assigns a value to a variable)
  - Transfer (reads an input data; writes a result)
  - Control (specifies which is the next step to be executed)
- Structured ….

Assignment

- Aim: give a value to a variable
- Description:

v ← <expression>

Rmk: sometimes we use := instead of ←

- Expression = syntactic construction used to describe a computation

It consists of:

  - Operands: variables, constant values
  - Operators: arithmetical, relational, logical

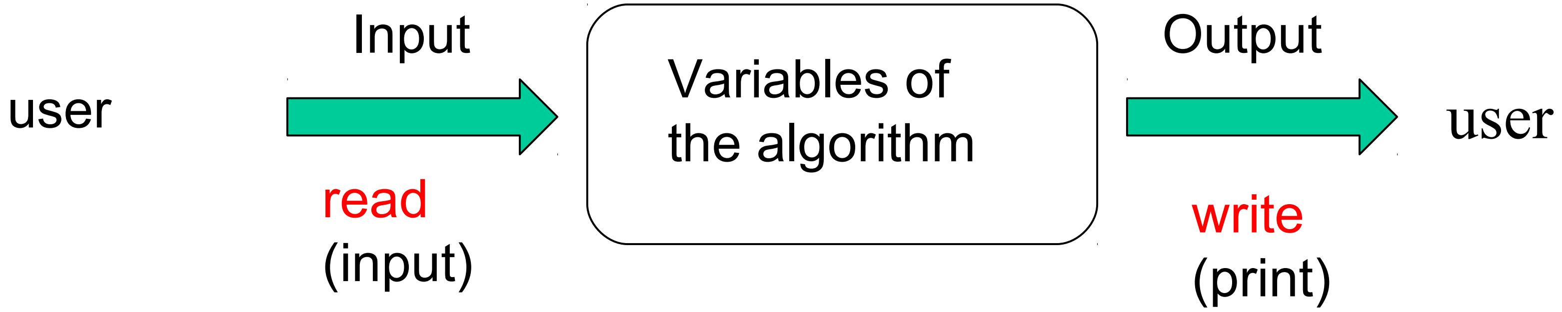
Operators

- Arithmetical:
  + (addition), - (subtraction), *(multiplication),
  / (division), ^ (power),
  DIV (from divide) or / (integer quotient),
  MOD (from modulo) or % (remainder)
- Relational:
  = (equal), != (different),
  < (less than), <= (less than or equal),
  >(greater than) >= (greater than or equal)
- Logical:
  OR (disjunction), AND (conjunction), NOT (negation)

Input/Output

- Aim:
  - read input data
  - output the results
- Description:

read v1,v2,…	input v1, v2,…
write e1,e2,…	print e1, e2,…



user

Input

read

(input)

Variables of the algorithm

Output

write

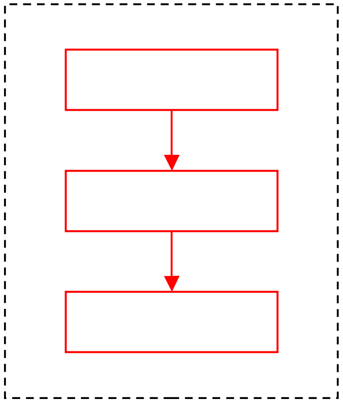
(print)

user

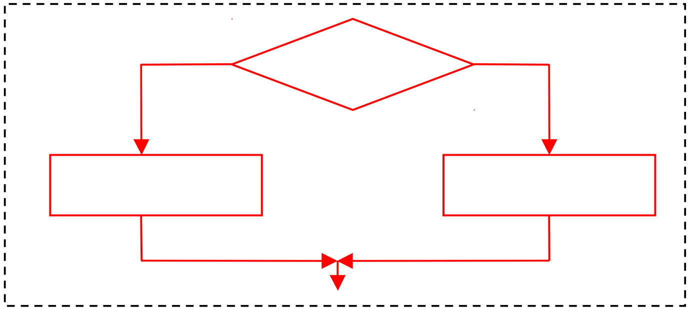
Instructions

Structured:

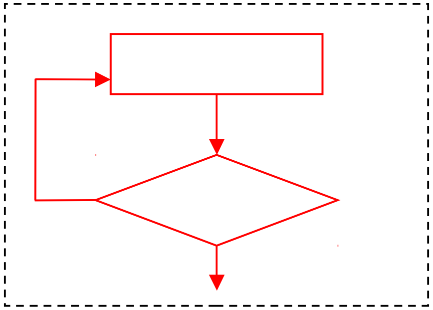
- Sequence of instructions



- Conditional statement

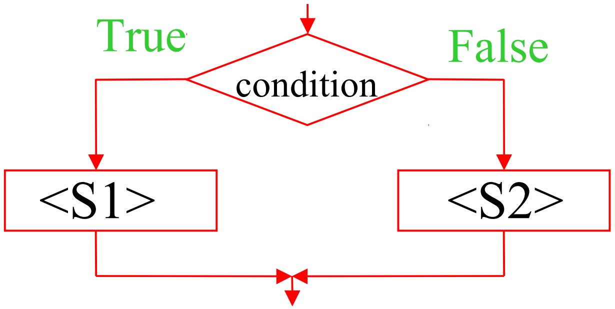


- Loop statement



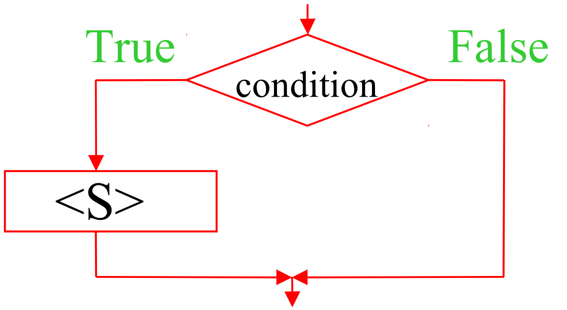
Conditional statement

- Aim: choosing between two or several alternatives depending on the value of some conditions



- General variant:

```
if <condition> then  <S1>
                     else  <S2>
endif
```



- Simplified variant:

```
if <condition> then <S>
endif
```

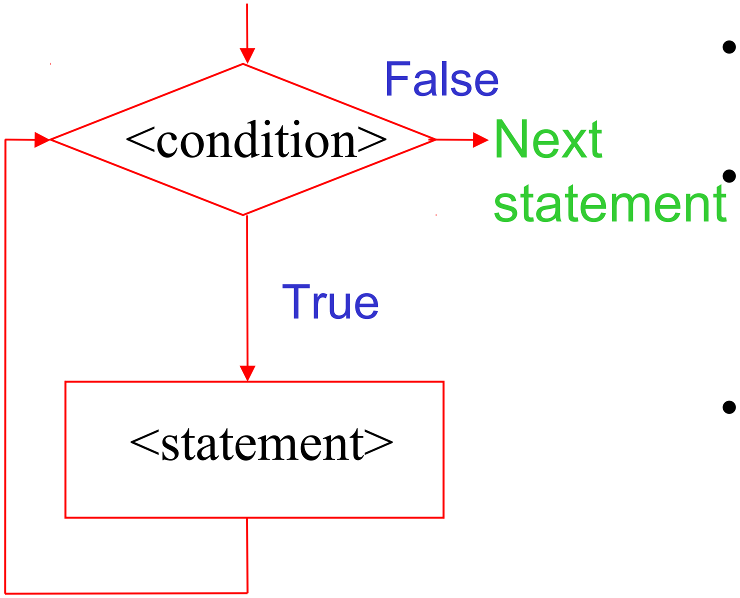
Loop statements

- Aim: repeating a processing step
- Example: compute a sum

  S= 1+2+…+i+…+n
- Characterized by:
  - Processing step which have to be repeated
  - Stopping (or continuation) condition

- Depending on the moment of analyzing the stopping condition there are two main loop statements:
  - Preconditioned loops (WHILE loops)
  - Postconditioned loops (REPEAT loops)

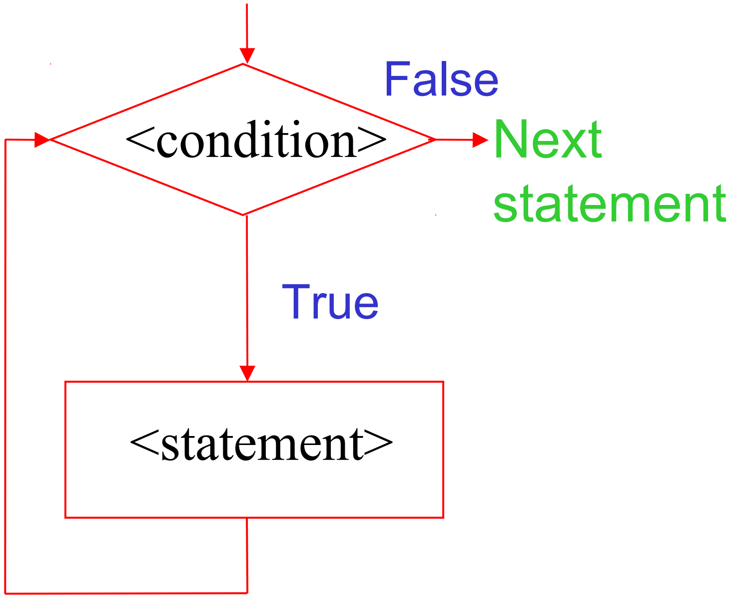
WHILE loop



```
while  <condition> do
       <statement>
endwhile
```

- First, the condition is analyzed
- If it is true then the statement is executed and the condition is analyzed again
- If the condition becomes false the control of execution passes to the next statement in the algorithm
- If condition never becomes false then the loop is infinite
- If the condition is false from the beginning then the statement inside the loop is never executed

WHILE loop



False

<condition>

Next statement

True

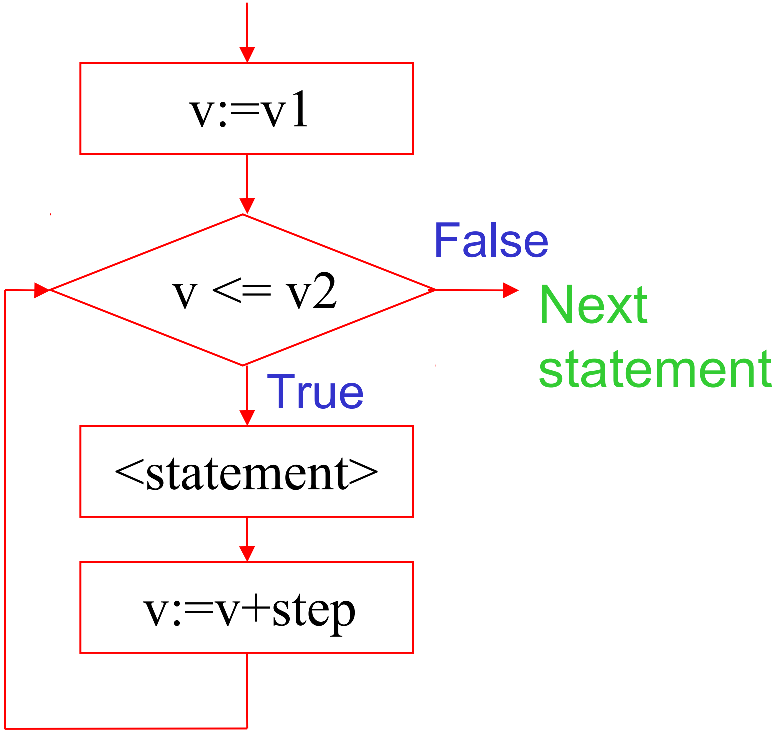
<statement>

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$$

```
while  <condition> do
           <statement>
endwhile
```

```
S:=0   // initialize the variable which will
       //  contain the result
i:=1   // index intialization
while i<=n do
   S:=S+i  // add the current term to S
   i:=i+1  // prepare the next term
endwhile
```

FOR loop



v:=v1

v <= v2

False

Next statement

True

<statement>

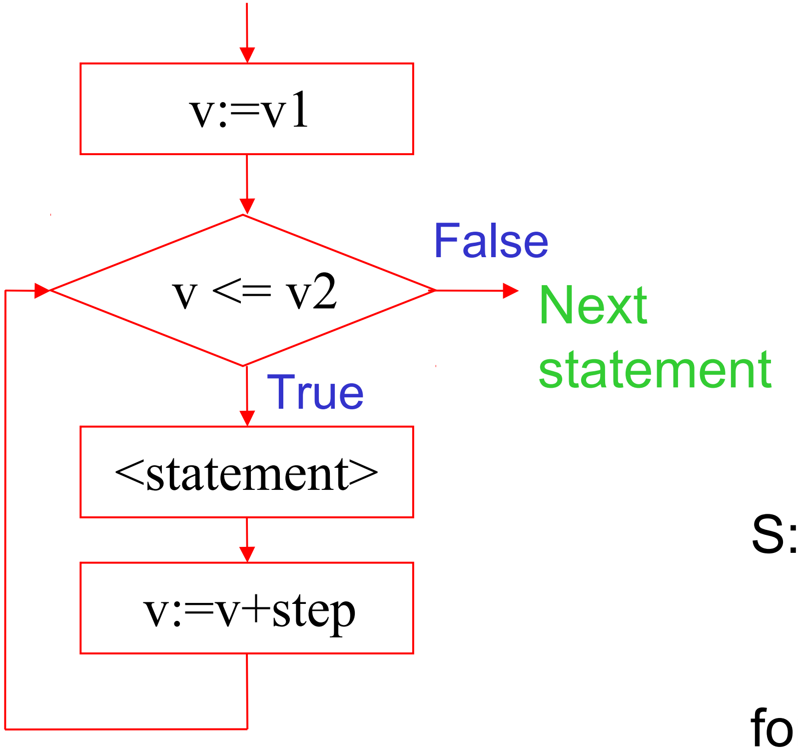
v:=v+step

- Sometimes the number of repetitions of a processing step is known apriori
- Then we can use a counting variable which varies from an initial value to a final value using a step value
- Repetitions: v2-v1+1 if step=1

```
for  v:=v1,v2,step do
             <statement>
endfor
```

```
v:=v1
while v<=v2 do
          <statement>
          v:=v+step
endwhile
```

FOR loop



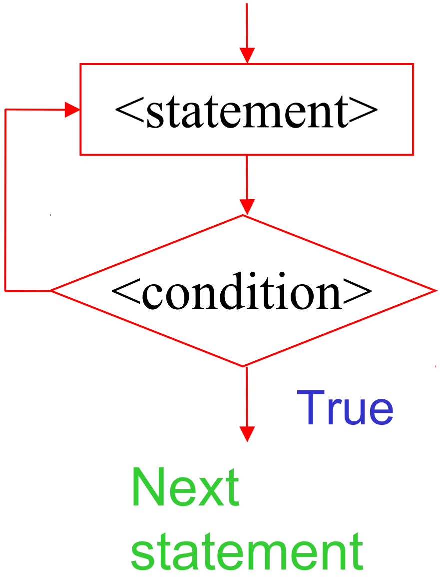
$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$$

```
for  v:=v1,v2,step do
              <statement>
endfor
```

```
S:=0   // initialize the variable which will
       //  contain the result

for i:=1,n do
   S:=S+i  // add the term to S
endfor
```

REPEAT loop



<statement>

<condition>

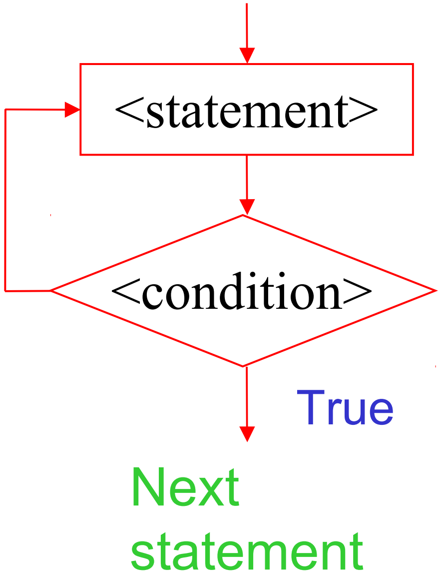
True

Next statement

```
repeat <statement>
until <condition>
```

- First, the statement is executed. Thus it is executed at least once
- Then the condition is analyzed and if it is false the statement is executed again
- When the condition becomes true the control passes to the next statement of the algorithm
- If the condition doesn’t become true then the loop is infinite

REPEAT loop



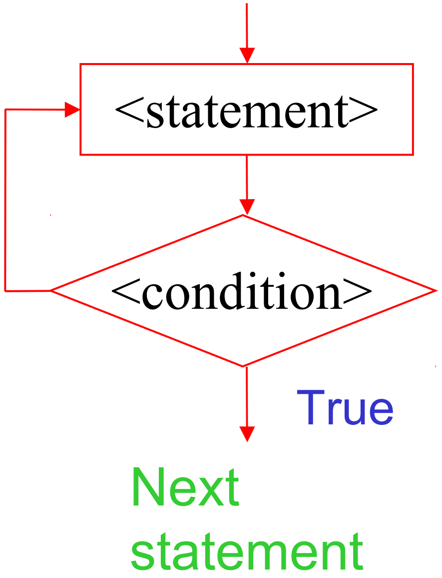
repeat <statement>
until <condition>

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$$

```
S:=0
i:=1
repeat
   S:=S+i
   i:=i+1
until i>n
```

```
S:=0
i:=0
repeat
   i:=i+1
   S:=S+i
until i>=n
```

REPEAT loop



<statement>

<condition>

True

Next statement

```
repeat <statement>
until <condition>
```

Any REPEAT loop can be transformed in a WHILE loop:

```
<statement>
while NOT <condition> DO
    <statement>
endwhile
```

Summary

- Algorithms are step-by-step procedures for problem solving

- They should have the following properties:
    - Generality
    - Finiteness
    - Non-ambiguity (rigorousness)
    - Efficiency

- Data processed by an algorithm can be
    - simple
    - structured (e.g. arrays)

- We describe algorithms by means of pseudocode

Summary

- Pseudocode:

Assignment	:=
Data transfer	read (input), write (print)
Decisions	if … then … else … endif
Loops	while … do … endwhile for … do … endfor repeat … until

Example 1

Consider a table containing info about student results

No.	Name	Marks			ECTS	Status	Average
1	A	8	6	7	60		
2	B	10	10	10	60		
3	C	-	7	5	40		
4	D	6	-	-	20		
5	E	8	7	9	60		

Task: fill in the status and average fields such that

status = 1 if ECTS=60

status= 2 if ECTS belongs to [30,60)

status= 3 if ECTS<30

the average is computed only if ECTS=60

Example 1

The filled table should look like this:

No.	**Name**	**Marks**			**ECTS**	**Status**	**Average**
1	A	8	6	7	60	1	7
2	B	10	10	10	60	1	10
3	C	-	7	5	40	2	-
4	D	6	-	-	20	3	-
5	E	8	7	9	60	1	8

Example 1

What kind of data should we process ?

No.	Name	Marks			ECTS	Status	Average
1	A	8	6	7	60		
2	B	10	10	10	60		
3	C	-	7	5	40		
4	D	6	-	-	20		
5	E	8	7	9	60		

Input data: marks and ECTS

marks[1..5,1..3] : two dimensional array (matrix) with 5 rows and 3 columns

Pseudocode specification: integer marks[1..5,1..3]

Example 1

What kind of data should we process ?

No.	Name	Marks			ECTS	Status	Average
1	A	8	6	7	60		
2	B	10	10	10	60		
3	C	-	7	5	40		
4	D	6	-	-	20		
5	E	8	7	9	60		

Input data: marks and ECTS

ects[1..5] : one-dimensional array with 5 elements

Pseudocode specification: `integer ects[1..5]`

Example 1

What kind of data should we process ?

No.	Name	Marks			ECTS	Status	Average
1	A	8	6	7	60		
2	B	10	10	10	60		
3	C	-	7	5	40		
4	D	6	-	-	20		
5	E	8	7	9	60		

Output data: status and average

status[1..5], average[1..5] : one-dimensional arrays with 5 elements

Pseudocode specification:

```
integer status[1..5]
real average[1..5]
```

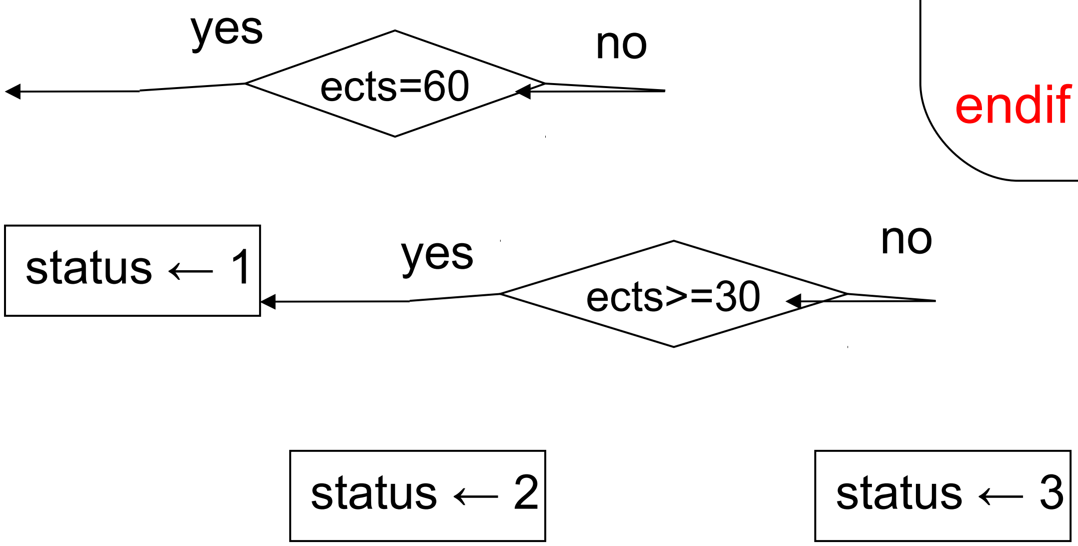
Example 1

Rule to fill in the status of a student

status = 1 if ECTS=60

status= 2 if ECTS belongs to [30,60)

status= 3 if ECTS<30



yes

ects=60

no

status ← 1

yes

ects>=30

no

status ← 2

status ← 3

Pseudocode description:

```
if ects=60 then status←1
   else if ects>=30 then status ← 2
            else status ← 3
          endif
endif
```

Python description

```
if ects==60:
          status=1
elif ects>=30:
          status=2
else:
          status=3
```

Example 1

Filling in the status of all students: for each student fill in the status field

Remark: Let us denote with n the number of students (in our example n=5)

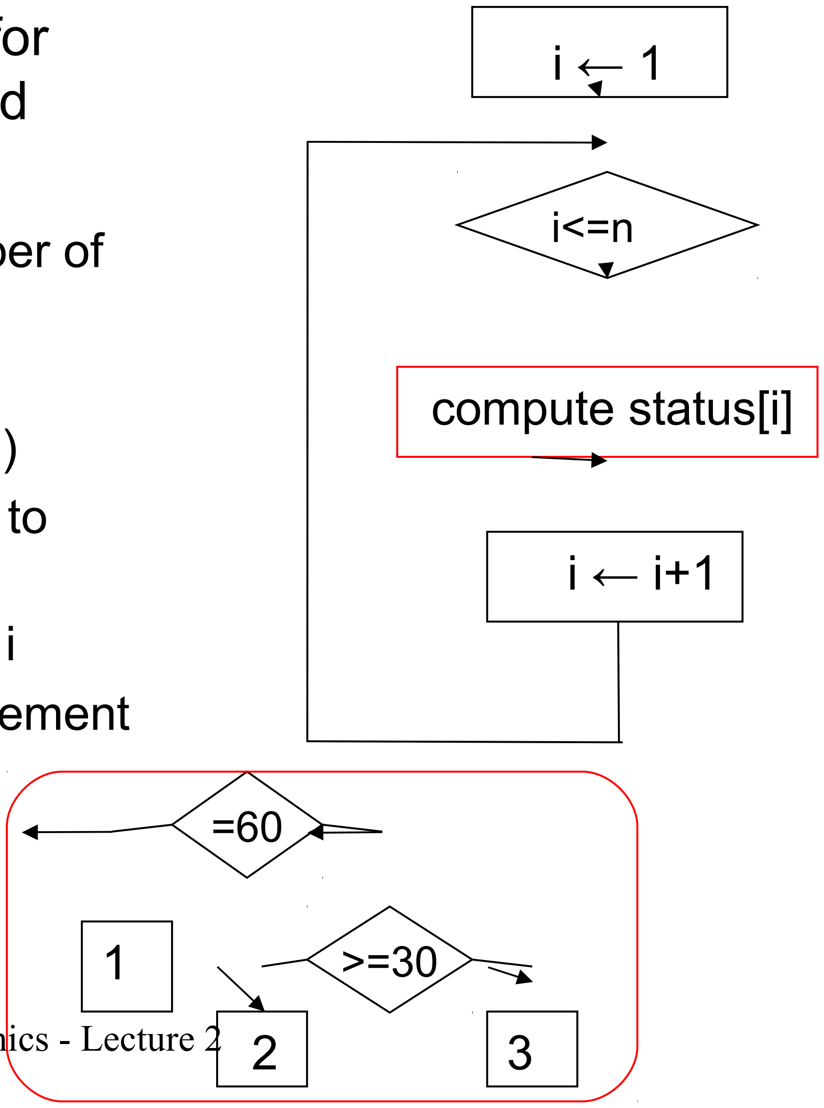
Step 1: start from the first element (i:=1)

Step 2: check if there are still elements to process (i<=n); if not then STOP

Step 3: compute the status of element i

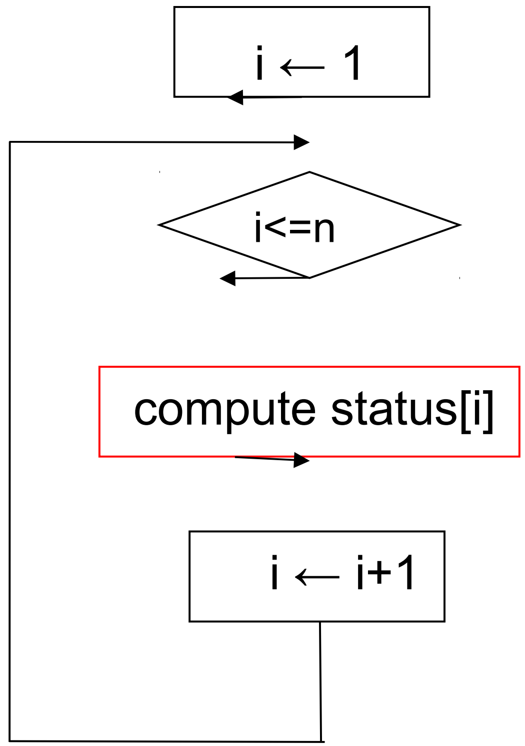
Step 4: prepare the index of the next element

Step 5: go to Step 2



Example 1

Filling in the status of all students: for each student fill in the status field



Pseudocode:

```
integer ects[1..n], status[1..n], i
i ← 1
while i<=n do
  if ects[i]=60 then status[i] ← 1
     else if ects[i]>=30 then status[i] ← 2
               else status[i] ← 3
            endif
  endif
  i ← i+1
endwhile
```

Example 1

Simplify the algorithm description by grouping some computation in "subalgorithms"

Pseudocode:

```
integer ects[1..n], status[1..n], i
i ← 1
while i<=n do
  status[i] ← compute(ects[i])
  i ← i+1
endwhile
```

Subalgorithm (function) description:

```
compute (integer ects)
  integer s
  if ects=60 then s ← 1
     else if ects>=30 then s ← 2
               else s ← 3
          endif
  endif
 return s
```

Remark: the subalgorithm describes a computation applied to generic data

Using subalgorithms

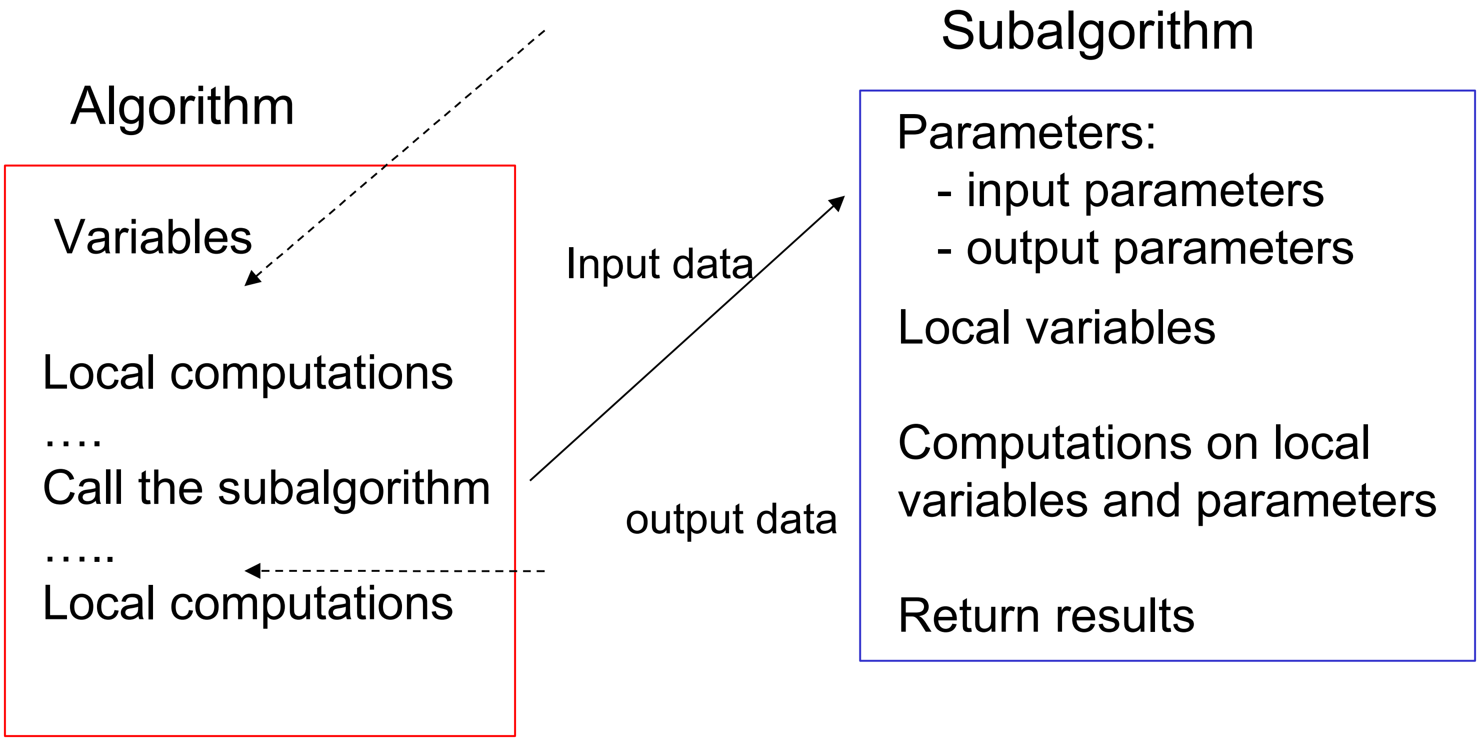
Basic ideas:

- Decompose the problem in subproblems
- Design for each subproblem an algorithm (called subalgorithm or module or function)
- The subalgorithm actions are applied to some generic data (called parameters) and to some additional data (called local variables)
- The execution of subalgorithm statements is ensured by calling the subalgorithm
- The effect of the subalgorithm consists of:
  - Returning some results
  - Modifying the values of some variables which are accessed by the algorithm (global variables)

Using subalgorithms

The communication mechanism between an algorithm and its subalgorithms:

- parameters and returned values



Subalgorithm

Algorithm

Variables

Local computations

….

Call the subalgorithm

…..

Local computations

Input data

output data

Parameters:

- input parameters
- output parameters

Local variables

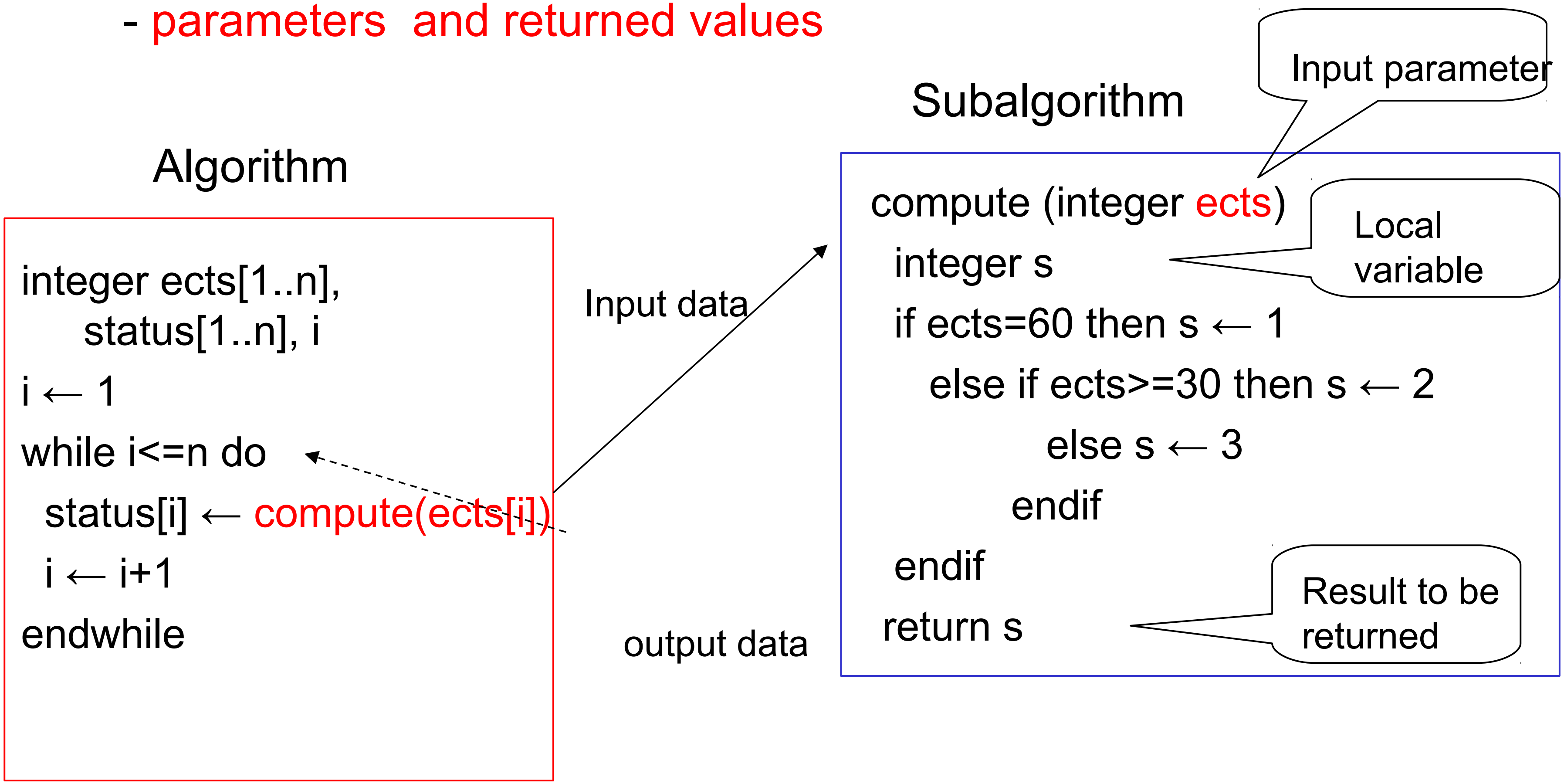
Computations on local variables and parameters

Return results

Using subalgorithms

The communication mechanism between an algorithm and its subalgorithms:

- parameters and returned values



Algorithm

```
integer ects[1..n],
    status[1..n], i
i ← 1
while i<=n do
  status[i] ← compute(ects[i])
  i ← i+1
endwhile
```

Input data

output data

Subalgorithm

```
compute (integer ects)
  integer s
  if ects=60 then s ← 1
     else if ects>=30 then s ← 2
              else s ← 3
          endif
  endif
 return s
```

Input parameter

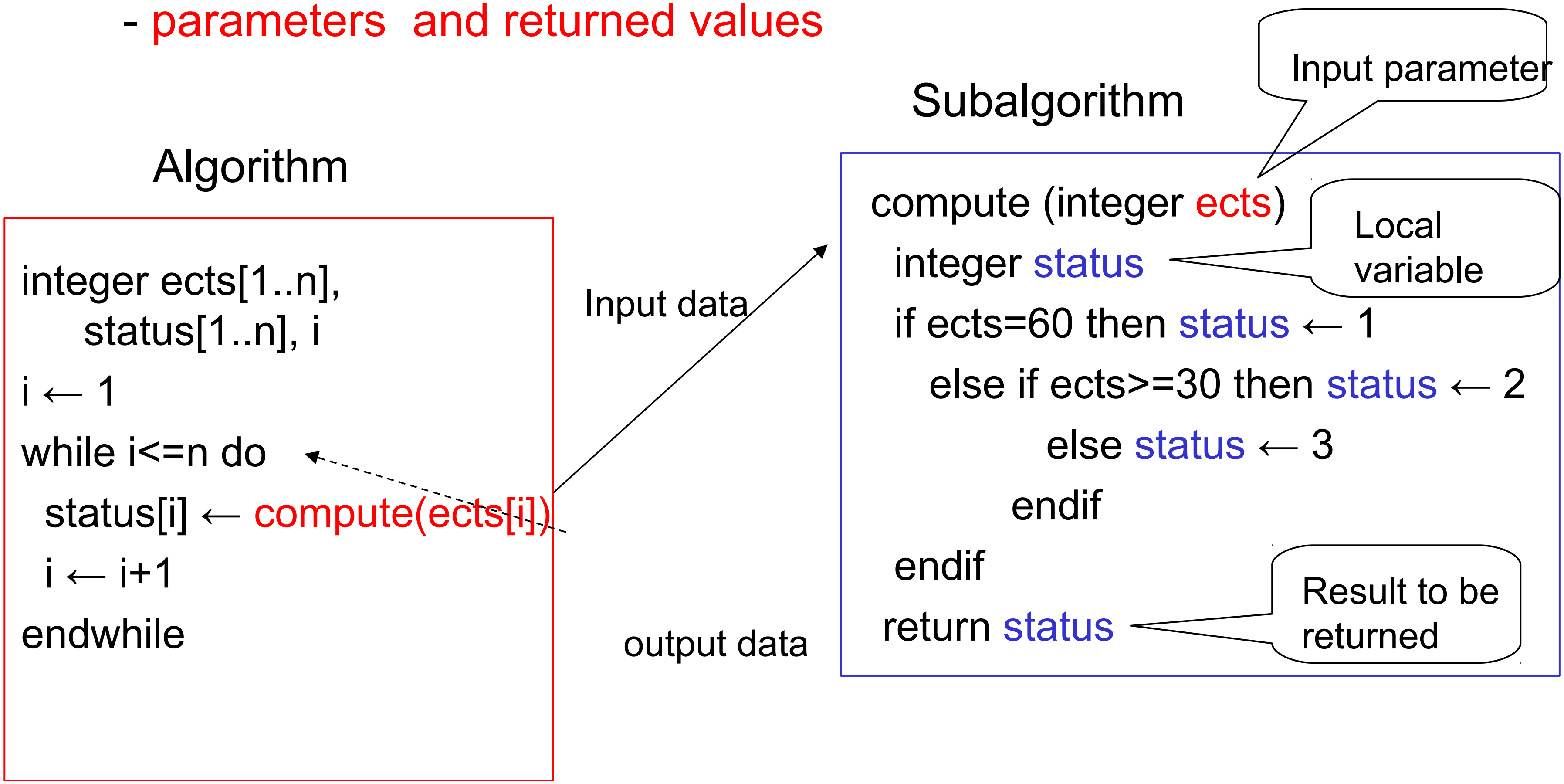
Local variable

Result to be returned

Using subalgorithms

The communication mechanism between an algorithm and its subalgorithms:

- parameters and returned values



Algorithm

```
integer ects[1..n],
    status[1..n], i
i ← 1
while i<=n do
  status[i] ← compute(ects[i])
  i ← i+1
endwhile
```

Input data

output data

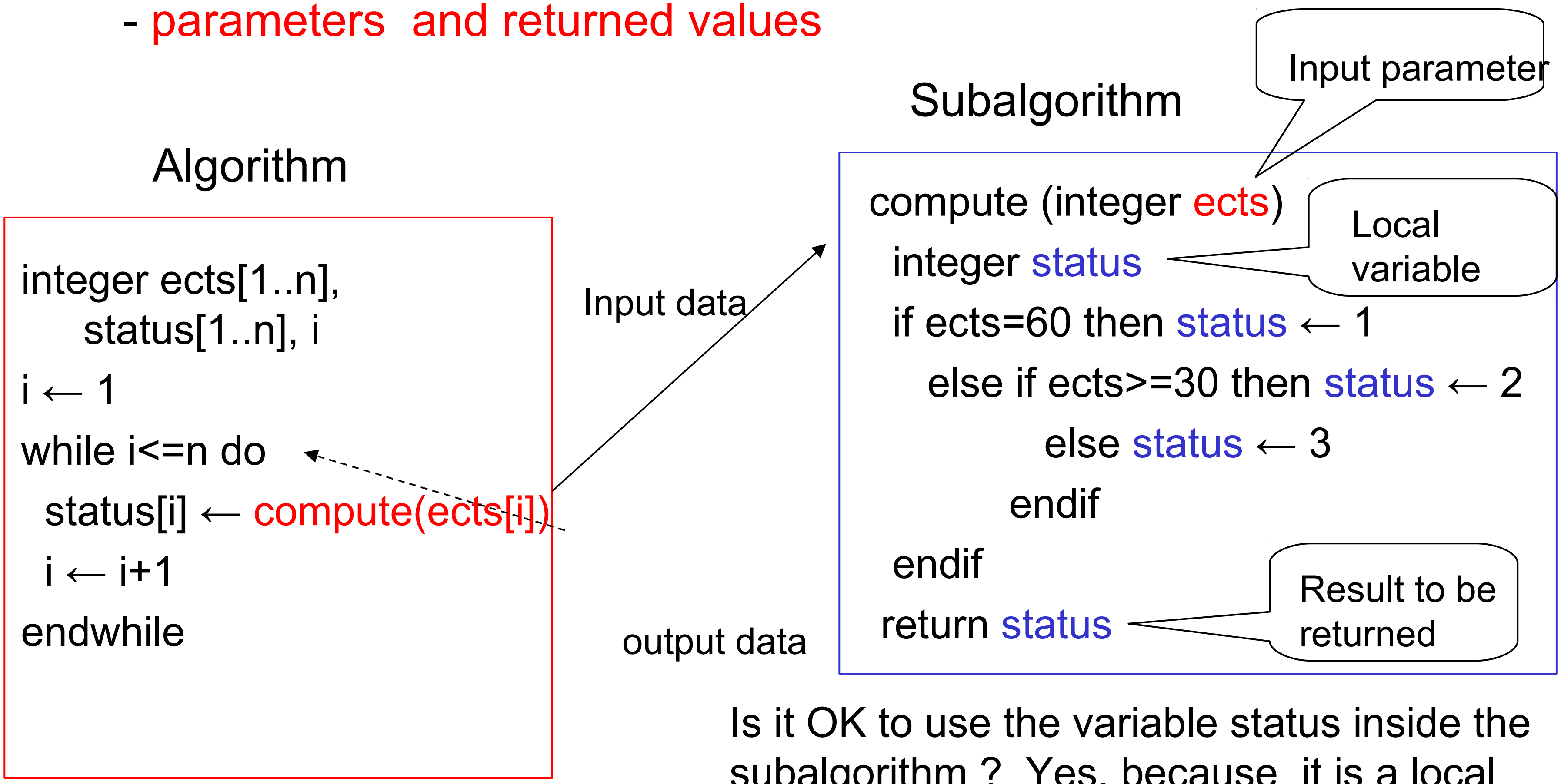
Subalgorithm

```
compute (integer ects)
  integer status
  if ects=60 then status ← 1
    else if ects>=30 then status ← 2
          else status ← 3
         endif
  endif
  return status
```

Using subalgorithms

The communication mechanism between an algorithm and its subalgorithms:

- parameters and returned values



Algorithm

```
integer ects[1..n],
    status[1..n], i
i ← 1
while i<=n do
  status[i] ← compute(ects[i])
  i ← i+1
endwhile
```

Input data

output data

Subalgorithm

```
compute (integer ects)
  integer status
  if ects=60 then status ← 1
     else if ects>=30 then status ← 2
            else status ← 3
          endif
  endif
  return status
```

Input parameter

Local variable

Result to be returned

Is it OK to use the variable status inside the subalgorithm ? Yes, because it is a local variable

Using subalgorithms

- Structure of a subalgorithm:

```
<subalgorithm name> (<formal parameters>)
    < declaration of local variables >
    < statements>
    RETURN <results>
```

- Call of a subalgorithm:

```
<subalgorithm name> (<actual parameters>)
```

Back to Example 1

Pseudocode:

```
integer ects[1..n], status[1..n], i
i:=1
while i<=n do
  status[i] ← compute(ects[i])
  i:=i+1
endwhile
```

Another variant

```
integer ects[1..n], status[1..n], i
for i:=1,n do
  status[i] ← compute(ects[i])
endfor
```

Subalgorithm (function) description:

```
compute (integer ects)
  integer status
  if ects=60 then status ← 1
     else if ects>=30 then status ← 2
               else status ← 3
          endif
  endif
 return status
```

Example 1: Python implementation

Python program:

```
ects=[60,60,40,20,60]
status=[0]*5
n=5
i=0
while i<n:
    status[i]=compute(ects[i])
    i=i+1
print status
```

Python function (module):

```
def compute(ects):
    if ects==60:
        status=1
    elif ects>=30:
        status=2
    else:
        status=3
    return status
```

Using a for statement instead of while:

```
for i in range(5):
    status[i]=compute(ects[i])
```

Remark: indentation is very important in Python

Example 1: computation of the average

Compute the averaged mark

```
integer marks[1..n,1..m], status[1..n]
real avg[1..n]
…
for i ← 1,n do
  if status[i]=1
    avg[i] ← computeAvg(marks[i,1..m])
  endif
endfor
```

Computation of an average

```
computeAvg(integer values[1..m])
real sum
integer i
sum ← 0
for i ← 1,m do
  sum ← sum+values[i]
endfor
sum ← sum/m
return sum
```

Example 1: computation of the average

Compute the averaged mark (Python example)

```
marks=[[8,6,7],[10,10,10],[0,7,5],[6,0,0],
    [8,7,9]]
status=[1,1,2,3,1]
avg=[0]*5

for i in range(5):
    if status[i]==1:
        avg[i]=computeAvg(marks[i])
print avg
```

Computation of an average (Python example)

```
def computeAvg(marks):
    m=len(marks)
    sum=0
    for i in range(m):
        sum = sum+marks[i]
    sum=sum/m
    return sum
```

Example 2 – greatest common divisor

Problem: Let a and b be to strictly positive integers. Find the greatest common divisor of a and b

Euclid’s method:

- compute r, the remainder obtained by dividing a by b
- replace a with b, b with r, and start the process again
- the process continues until one obtains a remainder equal to zero
- then the previous remainder (which, obviously, is not zero) will be the gcd(a,b).

Example 2 - greatest common divisor

How does this method work ?

1: $a=bq_1+r_1$, $0<=r_1<b$

2: $b=r_1q_2+r_2$, $0<=r_2<r_1$

3: $r_1=r_2q_3+r_3$, $0<=r_3<r_2$

…

i: $r_{i-2}=r_{i-1}q_i+r_i$, $0<=r_i<r_{i-1}$

…

n-1: $r_{n-3}=r_{n-2}q_{n-1}+r_{n-1}$, $0<=r_{n-1}<r_{n-2}$

n : $r_{n-2}=r_{n-1}q_n$, $r_n=0$

Remarks:

- at each step the dividend is the previous divisor and the new divisor is the old remainder
- the sequence of remainders is strictly decreasing, thus there exists a value n such that $r_n=0$ (the method is finite)
- using these relations one can prove that r_{n-1} is indeed the gcd

Example 2 - greatest common divisor

The algorithm
(WHILE variant):

```
integer a,b,dd,dr,r
  read a,b
  dd←a
  dr ← b
  r ← dd MOD dr
  while r<>0 do
      dd ← dr
      dr ← r
      r ← dd MOD dr
  endwhile
  write dr
```

The algorithm:
(REPEAT variant)

```
integer a,b,dd,dr,r
read a,b
dd ← a
dr ← b
repeat
      r ← dd MOD dr
      dd ← dr
      dr ← r
until r=0
write dd
```

Example 2 – gcd of a set of values

- Problem:

  Find the greatest common divisor of a sequence of non-zero natural numbers

- Example:

  gcd(12,8,10)=gcd(gcd(12,8),10)=gcd(4,10)=2

- Basic idea:

  compute the gcd of the first two elements, then compute the gcd between the previous gcd and the third element and so on …

  natural to use a (sub)algorithm for computing the gcd of two values

Example 2 – gcd of a set of values

- Structure of the algorithm:

```
gcd_sequence(INTEGER a[1..n])
INTEGER d,i
d ← gcd(a[1],a[2])
FOR i ← 3,n DO
  d ← gcd(d,a[i])
ENDFOR
RETURN d
```

```
gcd(integer a,b)
integer dd,dr,r
dd←a
dr ← b
r ← dd MOD dr
while r<>0 do
    dd ← dr
    dr ← r
    r ← dd MOD dr
endwhile
return dr
```

Example 3: The successor problem

Let us consider a natural number of 10 distinct digits. Compute the next number (in increasing order) in the sequence of all naturals consisting of 10 distinct digits.

Example: x= 6309487521

Next number consisting of different digits

6309512478

The successor problem

Step 1. Find the largest index i having the property that x[i-1]<x[i]

Example: x= 6309487521 i=6 (the pair of digits 4 and 8)

Step 2. Find the smallest element x[k] in x[i..n] which is larger than x[i-1]

Example: x=6309487521 k=8 (the digit 5 has this property)

Step 3. Interchange x[k] with x[i-1]

Example: x=6309587421 (this is a value larger than the first one)

Step 4. Sort x[i..n] increasingly (in order to obtain the smaller number satisfying the requirements)

Example: x=6309512478 (it is enough to reverse the order of elements in x[i..n])

The successor problem

Subproblems / subalgorithms:

Identify: Identify the rightmost element, x[i], which is larger than its left neighbour (x[i-1])

Input: x[1..n]
Output: i

Minimum: find the index of the smallest value in the subarray x[i..n] which is larger than x[i-1]
Input: x[i..n]
Output: k

Sorting: reverse the order of elements of the subarray x[i..n]
Input: x[i..n]
Output: x[i..n]

The successor problem

The general structure of the algorithm:

```
Successor(integer x[1..n])
integer  i, k
i←Identify(x[1..n])
if i=1
  then write "There is no successor !"
  else
      k ← Minimum(x[i..n])
      x[i-1]↔x[k]
      x[i..n] ← Reverse(x[i..n])
      write x[1..n]
endif
```

The successor problem

Identify the rightmost element, x[i], which is larger than its left neighbour (x[i-1])

```
Identify(integer x[1..n])
Integer i
i ← n
while (i>1) and (x[i]<x[i-1]) do
    i ← i-1
endwhile
return i
```

Find the index of the smallest value in the subarray x[i..n] which is larger than x[i-1]

```
Minimum(integer x[i..n])
Integer j
k ← i
for j ← i+1,n do
  if x[j]<x[k] and x[j]>x[i-1] then
     k ← j
return k
```

The successor problem

Reverse the order of elements of a subarray of x

```
reverse (integer x[left..right])
  integer i,j
  i ← left
  j ← right
  while i<j DO
      x[i]↔x[j]
      i ← i+1
      j ← j-1
  endwhile
  return x[left..right]
```

The successor problem

Python implementation:

```
def identify(x):
    n=len(x)
    i=n-1
    while (i>0) and (x[i-1]>x[i]):
        i=i-1
    return i

def minimum(x,i):
    n=len(x)
    k=i
    for j in range(i+1,n):
        if (x[j]<x[k]) and (x[j]>x[i-1]):
            k=j
    return k
```

```
def swap(a,b):
    aux=a
    a=b
    b=aux
    return a,b

def reverse(x,left,right):
    i=left
    j=right
    while i<j:
        x[i],x[j]=x[j],x[i]  # other type of swap
        i=i+1
        j=j-1
    return x
```

The successor problem

Python implementation:

```
x=[6,3,0,9,4,8,7,5,2,1]
print “Digits of the initial number :",x
i=identify(x)
print "i=",i
k=minimum(x,i)
print "k=",k
x[i-1],x[k]=swap(x[i-1],x[k])
print “Sequence after swap:",x
x=reverse(x,i,len(x)-1)
print “Sequence after reverse:",x
```

Summary

- The problems are usually decomposed in smaller subproblems solved by subalgorithms
- A subalgorithm is characterized through:
  - A name
  - Parameters (input data)
  - Returned values (output data)
  - Local variables (additional data)
  - Processing steps
- Call of a subalgorithm:
  - The parameters values are set to the input data
  - The statements of the subalgorithm are executed

Next lecture will be on …

- how to verify the correctness of an algorithm
- some formal methods in correctness verification