Case Study: concordance problem

- GIVEN: text (i.e. a sequence of *words*).
- TO DO: parse the text and discover the words. For each word examine if this its first occurrence.
- If this is the case memorize it.
- Otherwise increment a counter associated to the number of occurrences of the given word.
- Similar issues encountered in analyzing natural language (natural language processing). *What is the text about ?* *Statistical language processing*.
- Also in compilers.
- First phase in a compiler: lexical analysis.
- Identifies words. Represents program by *lexical tokens*.
- Example: x = 5; [IDENT "x"] [EQL] [NUMBER 5] [SEMICOLON].
- To represent x: <identifier>. Also: pointer to a list (table) of identifiers.

Solution

- Create a linked list of words.
- For each new word:
- *If not found in the list add it.*
- *Otherwise increment a counter associated to the given word.*

Implementation: interfaces

I am showing only more significant elements in the interfaces;

```
struct Token{
  std::string word;
  int count;
};
struct TokenNode{
  Token info;
  TokenNode * next;
};
class TokenNodeList{
public:
  TokenNodeList();
  void insertOrIncreaseCount (string);
private:
TokenNode *head;
TokenNode *tail;
}
```

Comments on implementation

- C++: a $struct$ is a class with all members public. It allows e.g. constructors. Though I don't recommend public data members, it's a better alternative to structs.
- Useful to maintain list in sorted order: to insert a new word we have to determine anyway that it doesn't appear in the list.
- Order on words: lexicographic order. The way words are listed in a dictionary: compare first letter first, then second letter, etc. Prefixes are smaller.
- $InsertOrIncreaseCount()$: search until you find element or larger one. If found increase count. Reuse code $insertInfoBefore()$, $insertInfo()$ (Course 3).

```
int compareStrings(string a, string b){
int i=0; int l1=a.length(); int l2=b.length();
for (int i=0;i<min(l1,l2);i++)
   if (a[i]<b[i])
       return 0;
    else
    if (a[i]>b[i])
       return 0;
return 1;
}
```

Implementing functions

```
void TokenList::insertOrIncreaseCount(string s){
  assert(!isEmpty());
  node *first=head;
  node *second = 0;
  while (lessThan(first->getInfo(),s))
  {
   second = first;
   first = first->getNextNode();
  };
  if (first->getInfo() == s)
  { // string found in list;
   first->count++;
  }else
     insertInfo(second,s);
}
```

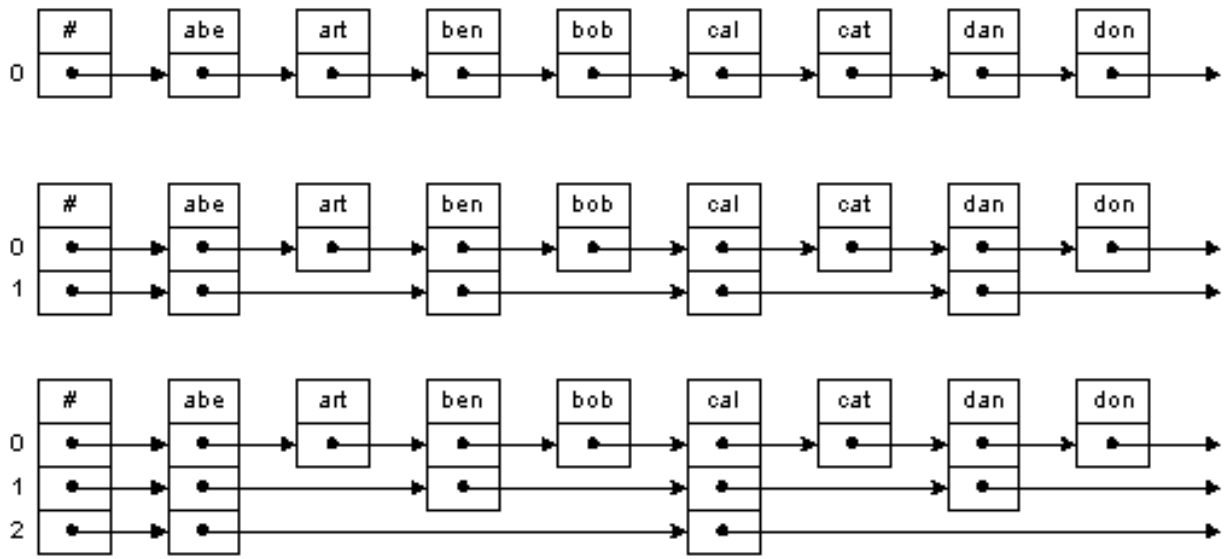
Concordance: main function

```
#include<fstream>

int main(int argc, char *argv[]) {
std::string nextItem;
TokenNodeList tl;
fstream file_op(argv[1],ios::in);

while (file_op >> nextItem)
  tl.insertOrIncreaseCount(nextItem);
// do something with token list
......
return 0;
}
```

Skip lists



Skip lists: implementation

- $k = 1, \ldots, \lfloor \log_2(n) \rfloor$, $1 \leq i \leq \lfloor n/2^{k-1} \rfloor - 1$.
- Item $2^{k-1} \cdot i$ points to item $2^{k-1} \cdot (i+1)$.
- every second node points to positions two node ahead,
- every fourth node points to positions four nodes ahead,
- every eigth node points to positions eigth nodes ahead,
- ....., and so on.
- **Different number of pointers in different nodes in the list !**
- half the nodes only one pointer.
- a quarter of the nodes two pointers,
- an eigth of the nodes four pointers,
- ....., and so on.
- $n \log_2(n)/2$ pointers.

Algorithm

- $n/2 \cdot 1 + n/4 \cdot 2 + n/8 \cdot 4 + \ldots$.
- Each product in the sum is $n/2$.
- How many terms in the sum ? $\lceil \log_2 n \rceil$. Total approximately $n \cdot \frac{\log_2(n)}{2}$.
- The number of pointers: the level of the node in the tree.
- Levels: *from 1 to* $\lfloor \log_2(n) \rfloor + 1$.
- To search: first follow pointers on the higher level until a larger element is found or the list is exhausted.
- If a larger element is found, restart search from its predecessor, this time on a lower level.
- Continue doing this until element found, or you reach the first level and a larger element or the end of the list.

Pseudocode

```
find(element el){
p = the nonnull list on the highest level i;
while (el not found and i ≥ 0)
  if (p->key > el)
   p = a sublist that begins in the predecessor of p
   on the level −−i;
  else
   if (p->key < el)
     if p is the last element on the level i
      p = a nonnull sublist that begins in p
      on the highest level < i;
      i = the number of this level;
     else
      p = p−>next;
}
```

Inserting and deleting nodes

- Problem: when inserting/deleting a node pointers of following nodes have to be restructured.
- Solution: rather than equal spacing, *random* spacing on a level.
- Number of nodes on each level approximately preserved.
- Level numbering: start with zero.
- New node inserted: probability 1/2 on first level, 1/4 second level, 1/8 third level, . . ., etc.
- Function $chooseLevel$: chooses the level of the new node.
- Generate random number. If in [0,1/2] level 1, [1/2,3/4] level 2, etc.
- Construct for "typical" case.
- Use randomness to simplify constructions.

Skip list node: interface

```
#define MAXLEVEL 4
class SkipListNode(){
public:
  SkipListNode(){}
  int key;
  SkipListNode ** next;
};
class SkipList{
public:
  SkipList();
  bool isEmpty() const;
  void choosePowers();
  int chooseLevel();
  int * SkipListSearch(int);
  void SkipListInsert(int);
private:
  typedef SkipListNode* nodePtr;
  nodePtr root[maxlevel];
  int powers[MAXLEVEL];
}
```

Skip lists: implementation

```
SkipList::SkipList(){
for(int i=0;i<maxLevel;i++)
  root[i]=0;
}

bool SkipList::isEmpty() const{
return root[0]==0;
}
void SkipList::choosePowers(){
powers[maxLevel-1]= (int) pow(2,maxLevel-1)-1;
for(int i= maxLevel-2,j=0;i>=0;i--,j++)
  powers[i]= (int) pow(2,j+1);
}
int SkipList::chooseLevel(){
  int i,r=rand() % powers[maxLevel-1] + 1;
  for(i=1;i<maxLevel;i++)
   if (r< powers[i])
     return i-1;//return a level < than the highest level.
  return i-1; // return the highest level.
}
```

Skip lists: implementation (II)

```
nodePtr* SkipList::SkipListSearch(int key){
if (isEmpty()) return 0;
nodePtr prev,curr;
int lvl; //find the highest nonnull
for (lvl=maxLevel-1;lvl >= 0 && !root[lvl];lvl--)// level
prev=curr = root[lvl];
while (true){
  if (key == curr->key) // success if equal
   return curr;
  else if (key < curr->key) { // if smaller, go down
   if (lvl == 0) // if possible
     return 0;
   else if (curr == root[lvl])
     curr = root[--lvl]; // by one level
   else curr = *(prev->next + --lvl); // starting from predecessor
  } // which can be the root
  else { // greater
```

Skip lists: implementation (III)

```
  else { // greater
   prev = curr;
   if (*(curr->next+lvl) != 0) // go to the next nonnull node
     curr = *(curr->next+lvl); // on the same level
   else { // or a list on a lower level
     for(lvl--; lvl>=0 & & *(curr->next + lvl)==0;lvl--)
     if (lvl == 0)
      curr = *(curr->next+lvl);
     else return 0;
     }
   }
  }
}
void SkipList::skipListInsert(int key){
nodePtr curr[maxLevel], prev[maxLevel],newNode;
int lvl, i;
curr[maxLevel-1]=root[maxLevel-1];
prev[maxLevel-1]=0;
```

Skip lists: implementation (IV)

```
for(lvl=maxLevel-1;lvl >=0;lvl--){
  while (curr[lvl] &&curr[lvl]->key < key){
   prev[lvl] = curr[lvl];
  }
  if(curr[lvl] && curr[lvl]->key == key)// don't include
   return; // duplicates
  if (lvl > 0) // go one level down
   if (prev[lvl]==0){ // if not the lowest level,
     curr[lvl-1]=root[lvl-1]; // using a link
     prev[lvl-1]=0; // either from the root
   }
   else { // or from predecessor;
     curr[lvl-1]=*(prev[lvl]->next+lvl-1);
     prev[lvl-1]=prev[lvl];
   }
}
```

Skip lists: implementation (V)

```
lvl = chooseLevel(); //generate randomly level for node
newNode = new SkipListNode;
newNode->next = new nodePtr[sizeof(nodePtr)*(lvl+1)];
newNode->key = key;
for (i=0; i<= lvl;i++){ // initialize next fields of
  *(newNode->next +i) = curr[i]; //newNode and reset to newNode
  if (prev[i] == 0) // either fields of the root
   root[i] = newNode; // or next fields of newNode's
  else *(prev[i]->next +i)= newNode; // predecessors
  }
}
```

Self-organizing lists

- Singly/doubly linked lists: sequential search to find element.
- If we know what is likely to be searched next one can improve efficiency of search by dynamically organizing the list. Depends on the stream of data.
- Move to front: after desired element located, put it at the beginning of the list. Assume that recently seen elements are more likely to be searched again.
- Transpose method: after desired element located, transpose it with previous element (unless at the head).
- Count method: Order the list by the number of times elements are accessed.
- Ordering method: order the list using criteria natural for the information under scrutiny (alphabetical).
- First three methods: new information added at the end. Last one: sorted order.

Comparing efficiency of ordering methods

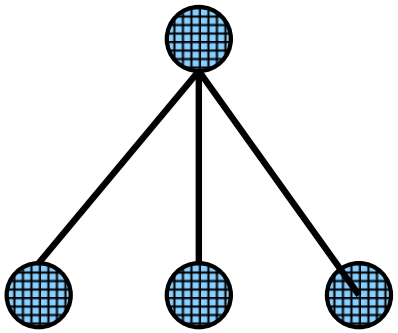
- Metric: number of comparisons/maximum number of comparisons.
- Usually: with optimal static ordering. All data ordered by frequency of occurrence. Requires two passes.
- Example: text; sum of combined lengths is 46. MTF: 33 comparisons (71.7%), plain search 30(62.5%).
- four benchmarks. second, fourth: computer programs. rest: English language text.
- Count, MTF twice as costly as optimal static ordering.
- of course, table only comparison, not other operations (e.g. pointer movement).

$\frac{Different}{All}$	$\frac{156}{347}$	$\frac{149}{423}$	$\frac{609}{1{,}510}$	$\frac{550}{2{,}847}$	$\frac{1{,}163}{5{,}866}$	$\frac{2{,}013}{23{,}065}$
$Optimal$	28.5	26.4	24.5	17.6	16.2	10.0
$Plain$	70.3	71.2	67.1	56.3	51.7	35.4
MTF	61.3	49.5	54.5	31.3	30.5	18.4
$Trs.$	68.8	69.5	66.1	53.3	49.4	32.9
$Count$	61.2	51.6	54.7	34.0	32.0	19.8
$Alph.$	50.9	45.6	48.0	55.7	50.4	50.0

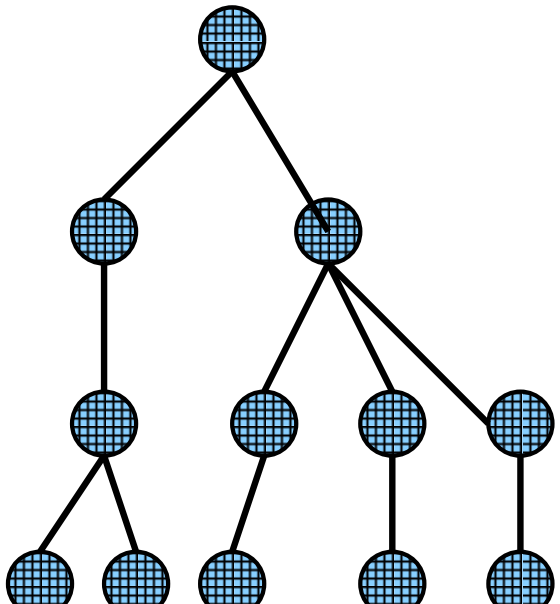
Binary trees

- A tree consists of *nodes* and *arcs*.
- Depicted upside down with its root at the top and the leaves at the bottom.
- Can be defined as follows:
- An empty structure is a tree.
- If $t_1, \ldots, t_k$ are disjoint trees, then the structure whose root has as its children the roots of $t_1, \ldots, t_k$ is also a tree.
- Only structures generated by rules 1 and 2 are trees.

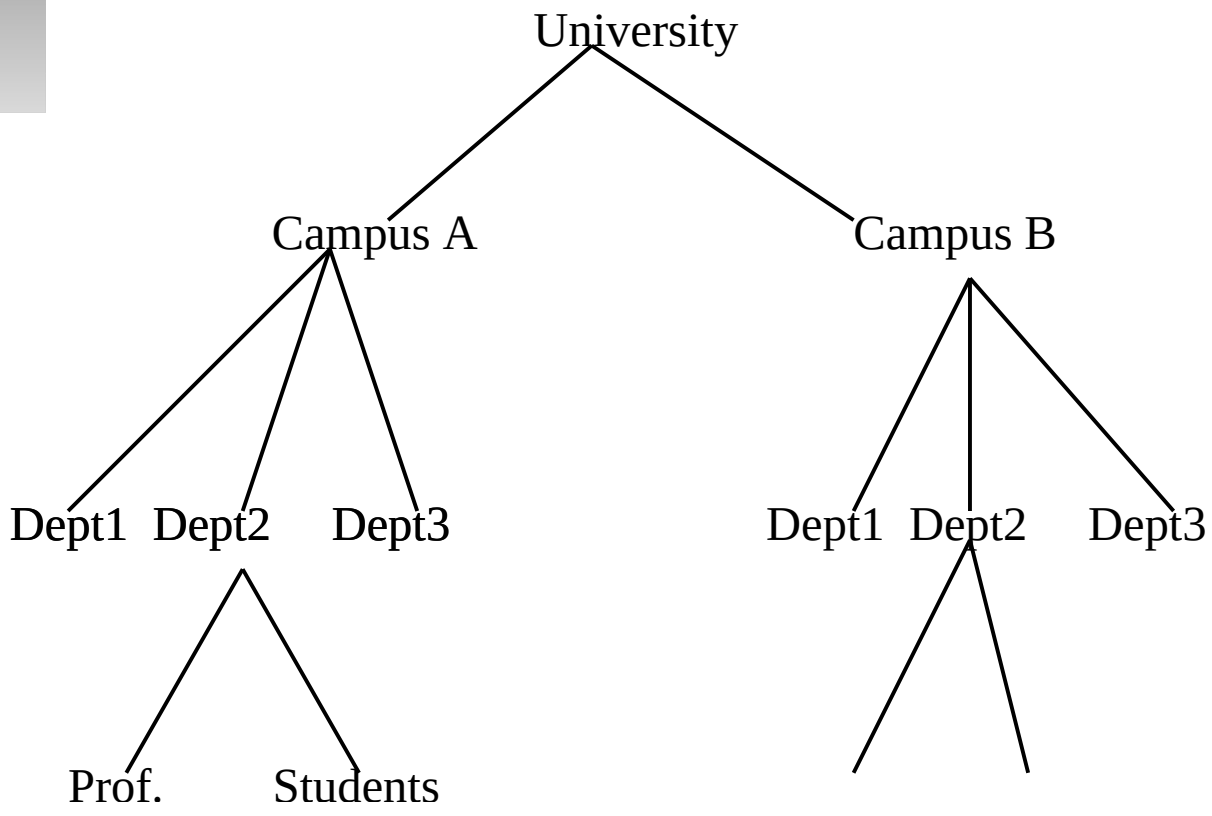
Examples of trees



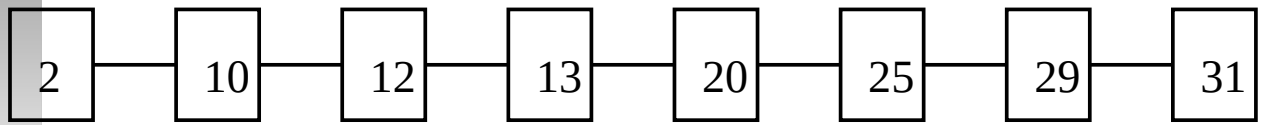
(a) (b) (c)

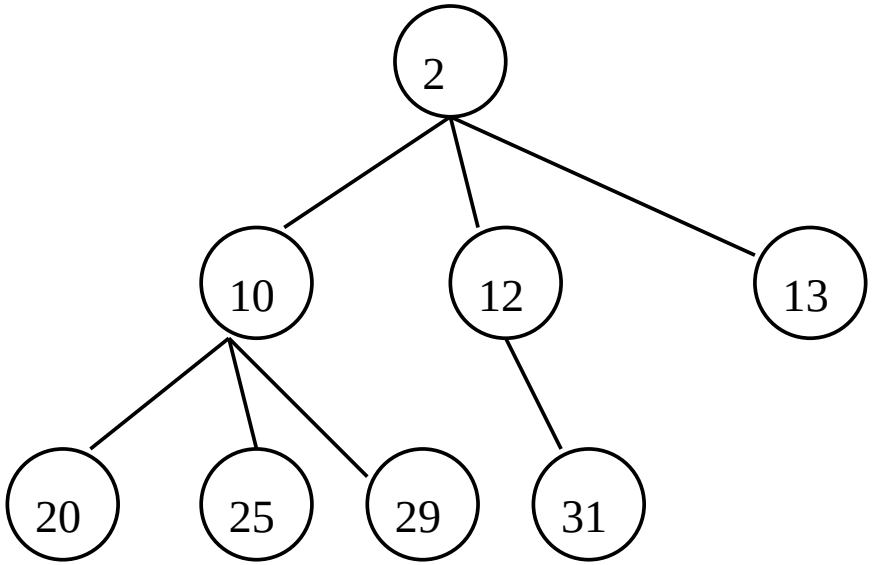


Examples of trees (II)



Linked lists vs. trees





Linked lists vs. trees

- Linked lists: search can take $\Omega(n)$ time. For large lists (and frequent search) this is significant.
- If transformed to *orderly* trees (trees with well-defined ordering criterion): can reduce this complexity.
- All trees discussed in this chapter: binary trees.
- Level of a node: 1 plus number of edges from root.
- Complete binary tree: 2^i nodes at level $i + 1$.

Binary search trees

- satisfy the following property:
- all nodes in a left subtree smaller than the root node.
- all nodes in right subtree larger.
- duplicate keys: avoided. Treated as *logical error*.
- Node keys: *scalar type* (allows comparisons).

Implementing binary search trees with vectors

- node is a structure with one information field and two "pointer" fields.
- pointer fields: indices of left and right subtrees (if they exist).
- can be inconvenient even for flexible arrays (vectors).
- "holes" in tree need to be eliminated. Populate array with special symbol (-1).
- also need to deal with cycles.

Example: BST with vector

Index	Info	Left	Right
{0}	13	4	2
{1}	31	6	-1
{2}	25	7	1
{3}	12	-1	-1
{4}	10	5	3
{5}	2	-1	-1
{6}	29	-1	-1
{7}	20	-1	-1

Implementing BST with pointers

- class BSTNode contains three fields:
- integer info,
- pointers left and right to left and right child.
- for easier traversal: also contain pointer to parent.
- all info: handled by class BST, which contains pointer to root.
- thus: members of BST node are public.

BSTNode: interface

```
class BSTNode{
public:
  BSTNode(){
   left = right = 0;
  }
  BSTNode(int el, BSTNode *l=0, BSTNode *r=0){
key = el; left = l; right = r;
}
int key;
BSTNode *left, *right;
};
```

Class BST: interface

```
class BST{
public:
BST(){
  root = 0;
}
~BST(){
  clear();
}
void clear(){
  clear(root); root=0;
}
bool isEmpty() const {
  return root == 0;
}
```

Class BST: interface (II)

```
void preorder(){
  preorder(root);
}
void inorder(){
  inorder(root);
}
void postorder(){
  postorder(root);
}
int *search(int el) const{
  return search(root,el);
}
void breadthFirst();
void iterativePreorder();
void iterativeInorder();
void iterativePostorder();
```

Class BST: interface (III)

```
void iterativePostorder();
void MorrisInorder();
void insert(int);
void deleteByMerging(BSTNode * &);
void findAnddeleteByMerging(int);
void deleteByCopying(BSTNode * &);
......
protected:
  BSTNode * root;
  void clear(BSTNode *);
  int *search(BSTNode *, int) const;
  void preorder (BSTNode *);
  void inorder (BSTNode *);
  void postorder (BSTNode *);
......
};
```

Searching a BST

```
BSTNode * BST::search(BSTNode *p,int el){
while (p != 0)
  if (el == p-> key)
   return p;
  else
   if (el < p->key)
       p = p-> left;
   else
       p = p-> right;
return 0;
}
```

Tree traversals

- Systematic visiting of all nodes in the tree.
- Each node visited once, does not specify order.
- Breadth-first traversal: visit each node, starting from the lowest level and moving down by level, visiting nodes from left to right.
- Depth-first traversals: go in subtree as deep as you can, backtrack. Differ on the order of visiting root, left, right subtrees.
- preorder: VLR.
- inorder: LVR.
- postorder: LRV.
- these definitions are recursive.

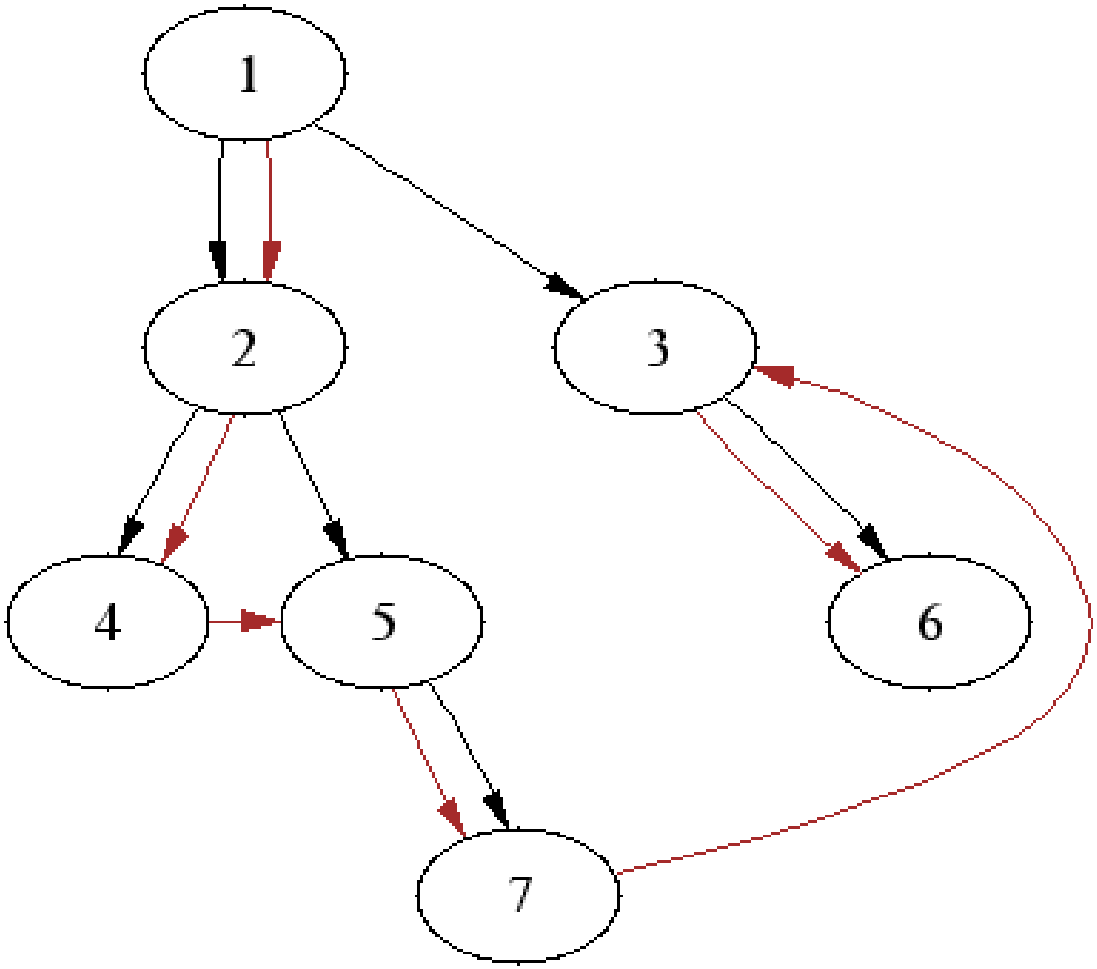
BFS traversal of a tree

```
void BST::breadthFirst(){
  Queue<BSTNode> q;
  BSTNode *p = root;
  if (p != 0){
   q.enqueue(p);
   while (!q.empty()){
       p = q.dequeue();
       visit(p);
       if (p-> left != 0)
         q.enqueue(p->left);
       if (p-> right != 0)
         q.enqueue(p->left);
   }
  }
}
```

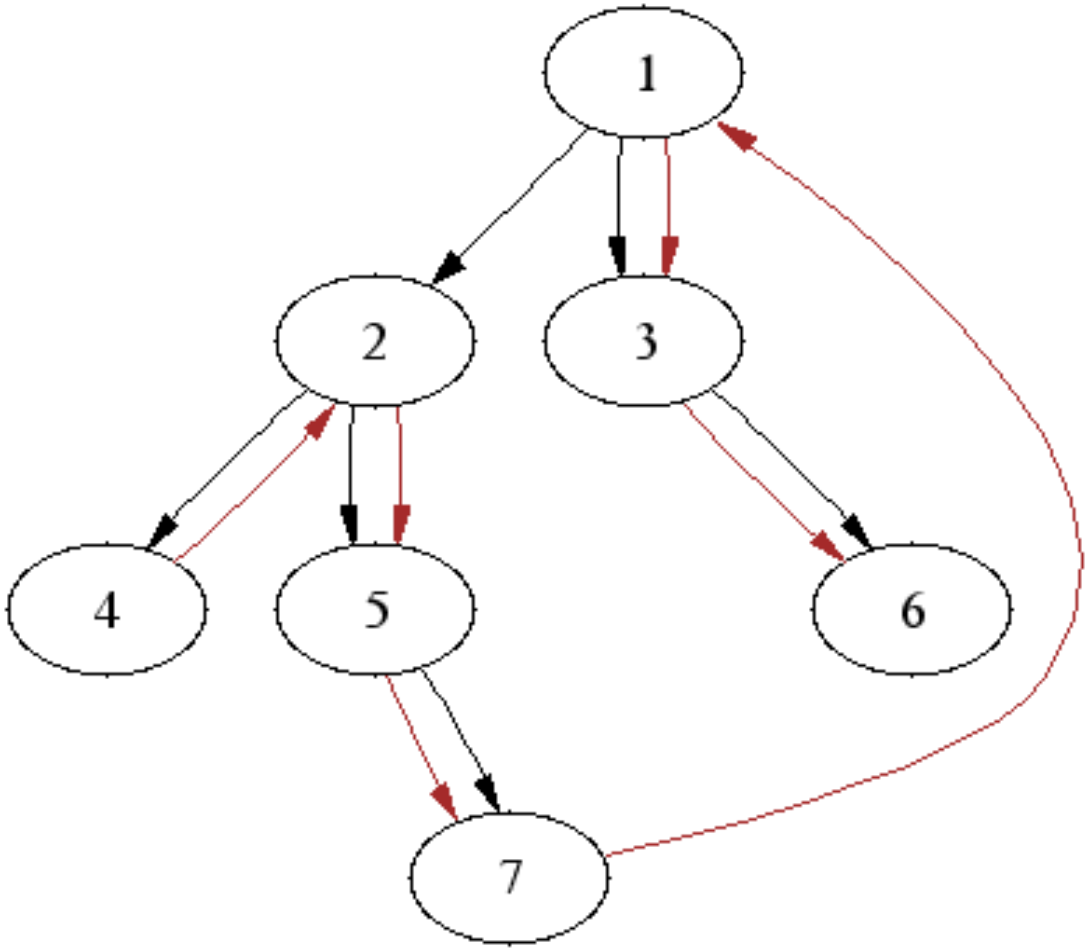
DFS traversals of a tree

```
void BST::inorder(BSTNode *p){
  if (p!=0){
   inorder(p->left);
   visit(p);
   inorder(p->right);
  }
}
void BST::preorder(BSTNode *p){
  if (p!=0){
   visit(p);
   preorder(p->left);
   preorder(p->right);
  }
}
void BST::postorder(BSTNode *p){
  if (p!=0){
```

Preorder traversal



Inorder traversal



Postorder traversal