# HOMEWORK 1

## October 15, 2012

Solve as many of the following problem as you can. Your writeup should reflect your own work !

You should send solutions by email to gabriel.istrate@gmail.com.

**DEADLINE: Saturday 20 October, 9 am (FIRM).**

# 1 Double counting applied to permutation identities

In the seminar we saw that one could prove identities using combinations by counting words in several ways.

Using a similar method show that when $a, b$ are integers greater or equal than two and $0 \leq k \leq a + b$ we have

$$C_{a+b}^k = \sum_{i=0}^{k} C_a^i \cdot C_b^{k-i}$$

(Hint: what kind of words does $C_n^k$ count ?)

# 2 Paths in the plane

You are in the plane at point P=(0,0) of the integer grid (think of a math notebook). You want to reach a point with coordinates $Q = (m, n)$, where $m, n$ are integers $\geq 1$. You want to go from $P$ to $Q$. At each step you either

- move one position up, or

- move one position to the right

(that is you "follow the grid" and never go left or down).

How many such paths do exist ? Justify your answer (a simple guess is not good enough).

# 3   Pseudocode: elementary school method for addition

Write the method we learned in school for adding two natural numbers in pseudocode.

# 4   Coin weighing: two fake coins

You have nine coins. *Two* of them are fake, and are lighter than the rest. You have a scale with two sides. You can put an arbitrary number of coins on each sides and observe if the two sides have equal weight or not, and which side is lighter.

Devise at least one algorithm and write it in pseudocode for detecting the two fake coins using at most *four* comparisons.

Extra Credit: design TWO DIFFERENT algorithms.

# 5   Computing frequencies

An array contains n numbers (not necessarily distinct) in increasing order: $x[1] \leq x[2] \leq \ldots \leq x[n]$.

Give a pseudocode that will compute:

- the number of different numbers in the array.

- for each of them the number of times it appears in the array.